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Convergence to Nonlinear Diffusion Waves for Solutions of a System of Hyperbolic Conservation Laws with Damping

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Abstract. We consider a model of hyperbolic conservation laws with damping and show that the solutions tend to those of a nonlinear parabolic equation time-asymptotically. The hyperbolic model may be viewed as isentropic Euler equations with friction term added to the momentum equation to model gas flow through a porous media. In this case our result justifies Darcy's law time-asymptotically. Our model may also be viewed as an elastic model with damping.

1. Introduction

Consider the following hyperbolic conservation laws with damping

$$\begin{aligned} &v_t - u_x = 0, \\ &u_t + p(v)_x = -\alpha u, \quad \alpha > 0, \quad p'(v) < 0. \end{aligned} \tag{1.1}$$

The system may be viewed as isentropic Euler equations in the Lagrangian coordinates with friction term $-\alpha u$ for the momentum equation. Thus it models the compressible flow through porous media. The commonly called porous media equation is obtained by approximating the second equation with Darcy's law

$$\begin{aligned} v_t &= \frac{-1}{\alpha} p(v)_{xx}, \\ p(v)_x &= -\alpha u. \end{aligned} \tag{1.2}$$

The purpose of this paper is to show that Darcy's law may be obtained from the more complete equations (1.1) time-asymptotically. That is, solutions of (1.1) tend

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