

Multi-Parameter Quantum Groups Related to Link-Diagrams[★]

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Received March 29, 1991

Abstract. We apply the Faddeev–Reshetikhin–Taktajan method for the construction of Quantum Groups to the Yang–Baxter matrices which are related to the invariants of oriented links in $\Sigma \times [0, 1]$, where Σ is a non-trivial 2-dimensional surface. We obtain multi-parameter ribbon Hopf algebras that differ in many respects from their one-parameter counterparts. Among the main differences we mention the existence of a non-central quantum determinant and the fact that the number of independent generators is higher than in the one-parameter case.

1. Introduction

It has been pointed out that invariants of links in the 3-dimensional space $\Sigma \times [0, 1]$, where Σ is a (open or closed) 2-dimensional surface, can be conveniently described by an algebraic structure defined on the set of oriented link-diagrams on Σ ([1, 2]).

In order to define such an algebraic structure and construct the invariants, we associate to each edge of a given link-diagram an integer (label) in $\{1, \dots, N\}$; some properties being required for this labelling procedure (including a conservation (Kirchhoff) law for the labels of the four edges meeting at any given vertex). Furthermore we associate to each vertex a given function of the indeterminate variables x, x^{-1}, z, z^{-1} depending on whether v is of type L_+ or L_- (here we use the standard terminology of knot theory) and depending on the labels of the four edges meeting at v . In this way we are able to construct a partition function which defines the link-invariants [3].

[★] Supported in part by the Italian National Institute for Nuclear Physics (I.N.F.N.) and by the National Science Foundation (N.S.F.) under Grant DMS 89-01975 and Grant DMS/PHY 88-16214

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