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The Spectrum of the Algebra of Skew Differential Operators and the Irreducible Representations of the Quantum Heisenberg Algebra

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Abstract. The left spectrum of a wide class of the algebras of skew differential operators is described. As a sequence, we determine and classify all the algebraically irreducible representations of the quantum Heisenberg algebra over an arbitrary field.

Introduction

The role the Heisenberg algebra plays in mathematical physics (quantum mechanics, quantum field theory, cf. [J, C]) and representation theory (cf. [K, D]) is well known. In the theory of Kac–Moody algebras the Heisenberg algebra provides one of the major tools for construction of irreducible representations (cf. [FK]).

The development of the theory of quantum and classical integrable systems has lead to the notion of Quantum Lie Groups. These objects are certain Hopf algebras created by deformation of universal enveloping algebras of Lie algebras and algebras of functions on Lie groups (cf. [Ji, Dr, FRT, S]). It is natural to expect that the quantum Heisenberg algebra introduced in [FG], which is one of the principal actors of this work, is going to play the same role in the (not yet created) representation theory of quantum groups as the classical Heisenberg algebra does in the representation theory of conventional Lie groups.

The study of the quantum Heisenberg algebra involves naturally a more general class of algebras – the algebras of skew differential operators. These algebras are well known to specialists in ring theory. Here, however, they appear as the objects of noncommutative geometry; and our first concern is to investigate their *spectral* properties. The means of the investigation are provided by recently introduced *local noncommutative algebra* (cf. [R1, R2]).

The article is organized as follows.