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Positive Lyapunov Exponents for Schrödinger Operators with Quasi-Periodic Potentials

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Abstract. We present a new, simple way to estimate the rate of exponential growth (Lyapunov exponent) of solutions of the finite-difference Schrödinger equation:

$$((H-E)\psi)(n) \stackrel{\text{def}}{=} -[\psi(n+1)+\psi(n-1)]+[\lambda f(\alpha n+\theta)]\psi(n).$$

Here f is a non-constant real-analytic function of period 1 and α is irrational. For λ large we prove that the Lyapunov exponent is positive for every energy E in the spectrum of H and a.e. θ. In particular, the absolutely continuous spectrum of H is empty. In the continuum we study the quasi-periodic operator on $L^2(R)$

$$H=-\frac{d^2}{dx^2}-K^2[\cos x+\cos(\alpha x+\theta)]$$

for large K and show that for wide intervals of low energies the Lyapunov exponent is positive. The main idea, which originated from M. Herman's subharmonic argument [11], is to deform the phase θ to the complex plane. This enables us to avoid small denominator problems by moving them off the axis, making estimates much easier to perform. We recover the information for real θ using an elementary extension of Jensen's formula (subharmonicity).

Table of Contents