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Embedded Eigenvalues of Sturm Liouville Operators

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Abstract. In this work we study the behavior of embedded eigenvalues of Sturm-Liouville problems in the half axis under local perturbations. When the derivative of the spectral function is strictly positive, we prove that the embedded eigenvalues either disappear or remain fixed. In this case we show that local perturbations cannot add eigenvalues in the continuous spectrum. If the condition on the spectral function is removed then a local perturbation can add infinitely many eigenvalues.

Introduction

Let us consider selfadjoint realizations of differential expressions of the form

$$(lu)(x) = -u''(x) + q(x)u(x), \qquad x \in [0, \infty),$$

where q is a real valued, locally integrable function defined in $[0, \infty)$. The end point 0 is regular and we assume that the limit point case occurs at ∞. We are interested in studying the behavior of embedded eigenvalues when we add a function of compact support to the potential q.

In some way these local perturbations are very natural and the operators considered are very simple since they are second order ordinary differential operators. Nevertheless the behavior of embedded eigenvalues in this case is not yet completely understood.

The main hypothesis of this work is that the spectral function of the unperturbed operator should have a positive derivative. This prevents the occurrence of new eigenvalues and allow us to prove that the remaining eigenvalues cannot move.

Since we are using tools of the theory of ordinary differential equations our results hold only for one dimension. Under other assumptions on the potentials and for higher dimensions many interesting related results can be found in Agmon-Herbst-Skibsted [1].