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Random Walks in Asymmetric Random Environments

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Dedicated to Joel Lebowitz on his 60th birthday

Abstract. We consider random walks on $\mathbf{Z}^d$ with transition rates $p(x, y)$ given by a random matrix. If p is a small random perturbation of the simple random walk, we show that the walk remains diffusive for almost all environments p if $d>2$. The result also holds for a continuous time Markov process with a random drift. The corresponding path space measures converge weakly, in the scaling limit, to the Wiener process, for almost every p.

1. Introduction

Random walks are probably the most extensively studied models of non-equilibrium behaviour. On a lattice $\mathbf{Z}^d$, a random walk is defined by a matrix $p(x, y)$, $x, y \in \mathbf{Z}^d$ giving the probability of jumping from x to y at each time. The only constraints on p are

$$p(x, y) \geqq 0, \tag{1}$$

$$\sum_y p(x, y) = 1 \quad \forall x. \tag{2}$$

Usually one considers walks on *homogeneous environments*, which means that

$$p(x, y) = p(x-y). \tag{3}$$

For a random walk in a *random environment* (RWRE), p is a *random matrix*. The randomness models the effect of impurities on a physical system, and one would like to study properties of the walk (e.g. its long time asymptotics) for almost every sample p.

Apart from its obvious interest in the study of diffusion in non-homogeneous media, RWRE may be considered as a simple model related to various other physical situations. These include Anderson's tight-binding model for disordered

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