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Measure Solutions of the Steady Boltzmann Equation in a Slab

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Abstract. It is shown that the steady Boltzmann equation in a slab $[0, a]$ has solutions $x \to \mu_x$ such that the ingoing boundary measures $\mu_{0|\{\xi>0\}}$ and $\mu_{a|\{\xi<0\}}$ can be prescribed a priori. The collision kernel is truncated such that particles with small x-component of the velocity have a reduced collision rate.

1. Introduction

Throughout this paper, $v = (\xi, \eta, \zeta) \in \mathbb{R}^3$ will denote a velocity vector with x-, y- and z-components ξ, η and ζ respectively. x is the (one-dimensional) position in the interval $[0, a]$. This interval is also referred to as a "slab."

For two velocities $v, w \in \mathbb{R}^3$ and a collision parameter $n \in S^2$, we define the collision transformation

$$J: (v, n, w) \to (v', -n, w')$$

by

$$\begin{aligned} v' &= v - n(n, v-w), \\ w' &= w + n(n, v-w). \end{aligned} \tag{1.1}$$

Here, $(n, v-w)$ denotes the Euclidean inner product in $\mathbb{R}^3$. J is an involution ($J^2 = \mathrm{id}$) and preserves momentum and energy. It is also well-known (and easily checked) that $\|v'-w'\| = \|v-w\|$ and $|(n, v-w)| = |(n, v'-w')|$, so the collision kernel $B(n, v-w)$, which in effect only depends on $\|v-w\|$ and $|(n, v-w)|$, is invariant under the action of J.

We are concerned with the steady Boltzmann equation in the slab $0 \leqq x \leqq a$, for $f = f(x, v)$,

$$\xi \cdot \frac{d}{dx} f = C(f, f) \tag{1.2}$$