Commun. Math. Phys. 142, 261–283 (1991)

Communications in Mathematical Physics

Quantum Affine Algebras

Vyjayanthi Chari[1] and Andrew Pressley[2]

[1] School of Mathematics, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 5, India
[2] Department of Mathematics, King's College, Strand, London, WC2R 2LS, England, UK

Received August 7, 1990; in revised form March 18, 1991

Abstract. We classify the finite-dimensional irreducible representations of the quantum affine algebra $U_q(\hat{sl}_2)$ in terms of highest weights (this result has a straightforward generalization for arbitrary quantum affine algebras). We also give an explicit construction of all such representations by means of an evaluation homomorphism $U_q(\hat{sl}_2) \to U_q(sl_2)$, first introduced by M. Jimbo. This is used to compute the trigonometric R-matrices associated to finite-dimensional representations of $U_q(\hat{sl}_2)$.

1. Introduction

A quantum group is a Hopf algebra $U_q(\underline{a})$, depending on a parameter $q \in \mathbb{C}$, which "tends to" the universal enveloping algebra $U(\underline{a})$ of a Lie algebra $\underline{a}$ as q tends to 1. In this paper, we develop a highest weight theory for the finite-dimensional representations of $U_q(\underline{a})$ when $\underline{a}$ is the affine algebra $\hat{sl}_2$, assuming that q is not a root of unity. We also give a concrete construction of all finite-dimensional irreducible representations of $U_q(\hat{sl}_2)$. Many, but not all, of the results extend without difficulty to the case of $U_q(\hat{\underline{g}})$ with $\underline{g}$ any finite-dimensional complex simple Lie algebra.

As in the case of the quantum groups $U_q(\underline{g})$ [10], where there are 2^l irreducible representations of any given highest weight ($l = \operatorname{rank} \underline{g}$), the finite-dimensional irreducible representations of $U_q(\hat{sl}_2)$ are of 4 types depending on the choice of two signs. One of our main results (Theorem (3.5)) establishes a one-to-one correspondence between (isomorphism classes of) finite-dimensional irreducible representations of $U_q(\hat{sl}_2)$ of each type and polynomials with constant coefficient 1. A similar result was proved by Drinfel'd [4] for Yangians, which are deformations of $U(\underline{g}[t])$ (but in that case there is no question of signs).

In the classical case, the finite-dimensional irreducible representations of $\hat{\underline{g}}$ are constructed as follows [1]. One proves first that the centre of $\hat{\underline{g}}$ acts trivially on all such representations; thus, one is considering representations of the loop algebra