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Geometric Singularities and Spectra of Landau–Ginzburg Models

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Abstract. Some mathematical and physical aspects of superconformal string compactification in weighted projective space are discussed. In particular, we recast the path integral argument establishing the connection between Landau–Ginzburg conformal theories and Calabi–Yau string compactification in a geometric framework. We then prove that the naive expression for the vanishing of the first Chern class for a complete intersection (adopted from the smooth case) is sufficient to ensure that the resulting variety, which is generically singular, can be resolved to a smooth Calabi–Yau space. This justifies much analysis which has recently been expended on the study of Landau–Ginzburg models. Furthermore, we derive some simple formulae for the determination of the Witten index in these theories which are complimentary to those derived using semiclassical reasoning by Vafa. Finally, we also comment on the possible geometrical significance of *unorbifolded* Landau–Ginzburg theories.

1. Introduction

During the past year much progress has been made in understanding the previous mysterious connection between minimal model string vacua [1] and geometrical compactification on manifolds with trivial canonical bundle [2, 3]. The unifying link between the two is a Landau–Ginzburg description of the minimal model theories which gives rise, with little difficulty, to a nonlinear sigma model with Calabi–Yau target space. In particular, an orbifold of the Landau–Ginzburg theory lies in the same universality class as a string propagating in a Calabi–Yau background embedded in weighted projective space. Weighted projective space, unlike ordinary projective space, has quotient singularities. Hence, a generic Calabi–Yau manifold embedded in such a background has such singularities as well.