Commun. Math. Phys. 142, 193–216 (1991)

Communications in Mathematical Physics

A Proof That Witten's Open String Theory Gives a Single Cover of Moduli Space

Barton Zwiebach*

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Received December 30, 1990; in revised form March 29, 1991

Abstract. We show that Witten's open string diagrams are surfaces with metrics of minimal area under the condition that all nontrivial open Jordan curves be longer or equal to π. The minimal area property is used together with a mini-max problem to establish a new existence and uniqueness theorem for quadratic differentials in open Riemann surfaces with or without punctures on the boundaries. This theorem implies that the Feynman rules of open string theory give a single cover of the moduli of open Riemann surfaces.

1. Introduction and Summary

The open string field theory proposed by Witten [Wi] is based on a single interaction joining three open strings. This interaction gives a simple set of Feynman rules that build string diagrams. A string diagram is a Riemann surface with a choice of analytic coordinates at punctures. In open string theory the Riemann surfaces are surfaces with boundaries, and the punctures, that must lie on the boundary components, represent the external open strings. Typically, if we endow a Riemann surface with a suitable metric, it is possible to extract analytic coordinates around the punctures. This is the case for the string diagrams of [Wi], which are conveniently described in terms of metrics.

A necessary condition for the consistency of open string theory is that the Feynman rules construct string diagrams that provide a single cover of all relevant moduli spaces. While there is a fair amount of evidence [Wi, GMW, Gi, Og, BS, Sa], and it is widely believed to be the case, we lack a complete proof. This important point has been correctly emphasized by Samuel [Sa]. Not only we are missing a complete proof, but also a proper understanding of why it works. As we have learned in closed string theory, the fact that a set of Feynman rules work well at the classical level, namely, produce a single cover of moduli spaces of

* Supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under contract # DE-AC02-76ER03069