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A Boundary Value Problem Related to the Ginzburg–Landau Model

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This paper is dedicated to the memory of Michel Sirrugue

Abstract. We analyze the Ginzburg–Landau equation for a superconductor in the case of a 2-dimensional model: a cylindrical conductor with a magnetic field parallel to the axis. This amounts to find the extrema of the free energy

$$\mathscr{A}_\kappa = 1/2 \int_\Omega [|(\nabla - iA)\Phi|^2 + |B_A|^2 + \kappa/4(|\Phi|^2 - 1)^2]dx,$$

where Ω is a bounded domain with smooth boundary in $\mathbb{R}^2$, $A=(A_1, A_2)$ the vector potential, $B_A = \partial_1 A_2 - \partial_2 A_1$ the magnetic field, Φ a complex field. We describe the connected components of the maximal configuration space, i.e. of the set of all (A, Φ) with components in the Sobolev space $H^1(\Omega)$ and such that $|\Phi| = 1$ on the boundary, modulo the action of the gauge group. In the critical case $\kappa = 1$ we give a complete description of the minimal configurations in each component.

1. Introduction

The Ginzburg–Landau model [6] has been proposed in 1950 in order to give a phenomenological description of superconductivity. It is an experimental fact that at low temperatures and weak magnetic fields some materials get an infinite electric conductivity. This phenomenon has been described by Ginzburg and Landau with the help of a complex valued field Φ interacting with the magnetic field B. Φ and the vector potential A which generates B satisfy a system of nonlinear partial differential equations (see [6, 12 or 13]). The physical interpretation of Φ is rather complicated and was understood only after the appearance of the microscopic theory of Bardeen, Cooper and Schrieffer. Roughly speaking, this theory considers