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Bifurcation Frequency for Unimodal Maps

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Abstract. We consider some natural one-parameter unfoldings f_μ, of a unimodal map f_0 whose periodic points are hyperbolic and whose critical point is non-degenerate and eventually periodic. Among other facts, it follows from our theorems that, if the Julia set of f_0 does not contain intervals, the relative measure of the bifurcation set is zero at zero.

Introduction

It is extremely surprising that in such a simple space as an interval, there should exist important and rich dynamical systems. However many fascinating types of interval dynamics have been discovered. They are of interest in their own right as well as being useful mathematical models, and frequently as being part of higher dimensional systems.

Informally, we think of a dynamical system as a "system in movement;" as time goes by each point in a phase space evolves according to some deterministic law. An important feature of a dynamical system is its limit set; the set where the orbits accumulate. The dynamical behaviour inside the limit set can be "simple" or "complex." The results of this article support the view that systems with "simple" limit set are very frequent.

Here we deal with interval dynamics generated by iteration of unimodal maps. One simple case, the axiom A case, is when the periodic points are hyperbolic and the critical value lies inside the basin of a periodic sink. In this case we have a hyperbolic dynamic which is structurally stable and can be reduced to the dynamic of simpler symbolic models. There exist other cases in which the dynamics are described by absolutely continuous ergodic invariant probability measures.

We consider some natural one-parameter unfoldings f_μ ($\mu \geqq 0$), of a unimodal map f_0 whose periodic points are hyperbolic and whose critical point is eventually periodic. We have two cases depending on the topological structure of the Julia set Σ_0 of f_0 (the complement of the basin of the periodic sinks of f_0): if Σ_0 does not