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Equilibrium Statistics of a Vortex Filament with Applications

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Abstract. The thermodynamic functions and scaling exponents (including the Kolmogorov and Flory exponents) of a vortex filament in thermal equilibrium are calculated, giving a quantitative content to earlier qualitative analyses. The numerical results uncover a percolation property of vortex filaments near the maximum entropy state. The implications of the results for the onset of turbulence, for the structure of its inertial range, and for superfluid vortices are discussed. In particular, it is shown that vortex stretching pushes a vortex system to a polymeric state and a Kolmogorov spectrum.

Introduction

In an earlier paper [9] it was suggested that some properties of turbulent flow as well as of superfluid flow can be studied through the analysis of the statistical equilibria of vortex filaments. In particular, it was suggested that the inertial range of scales is in approximate thermal equilibrium, and it was shown that the vortex equilibrium of maximum entropy has a Kolmogorov spectrum. Other properties of turbulent vortices, in particular vortex folding, were also explained by an equilibrium model, and an analogy with an ansatz recently proposed in an XY model and in superfluidity was pointed out.

The goal of the present paper is to flesh out these ideas with a quantitative analysis of vortex equilibria. In particular, the energy and the entropy will be calculated as a function of the temperature and the vortex length, and the scaling exponents, in particular the Flory and Kolmogorov exponents for a filament, will be tabulated. A single filament will be considered, for reasons explained below. The calculations reveal an interesting percolation property of vortex filaments near the maximum entropy state that consists of "polymeric" configurations and has a

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