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Regularity of Constraints in the Minkowski Space Yang-Mills Theory*

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Abstract. For Yang-Mills theory in the Minkowski space it is proved that the constraint set is a smooth submanifold of the phase space consisting of square integrable Cauchy data.

1. Introduction

Yang-Mills equations can be rewritten as the evolution equations for the Cauchy data supplemented by the constraint condition. In the temporal gauge the evolution equations are

$$\dot{\mathbf{A}} = \mathbf{E}, \tag{1}$$

$$\dot{\mathbf{E}} = \operatorname{div}\mathbf{F} + [\mathbf{A};\mathbf{F}], \tag{2}$$

where

$$\mathbf{F} = d\mathbf{A} + [\mathbf{A},\mathbf{A}] \tag{3}$$

is the field strength of **A**, and

$$[\mathbf{A};\mathbf{F}]^a = c^a_{bc}\mathbf{A}^b_i\mathbf{F}^{ci} \tag{4}$$

is the Lie bracket in the Lie algebra $\mathfrak{g}$ of the structure group G of the theory, followed by the contraction in the spatial indices. The constraint equation is

$$\operatorname{div}\mathbf{E} + [\mathbf{A};\mathbf{E}] = 0. \tag{5}$$

In order to have an understanding of the theory sufficient for a subsequent quantization, we need to know the spaces which admit existence and uniqueness theorems for the system (1), (2), and (5), and the structure of the solution set of the constraint equation [since Eq. (5) is non-linear its solution set may have singularities].

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