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Finite-Dimensional Irreducible Representations of the Quantum Superalgebra $U_q[gl(n/1)]$

T. D. Palev¹ and V. N. Tolstoy²

 Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan and Institute for Nuclear Research and Nuclear Energy, Boul. Trakia 72, BG-1784 Sofia, Bulgaria*
² Institute of Nuclear Physics, Moscow State University, SU-119899 Moscow, USSR

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Abstract. It is shown that every finite-dimensional irreducible module over the general linear Lie superalgebra gl(n/1) can be deformed to an irreducible module of $U_q[gl(n/1)]$, a q-analogue of the universal enveloping algebra of gl(n/1). The results are extended also to all Kac modules, which in the atypical cases remain indecomposible. Within each module expressions for the transformations of the Gel'fand-Zetlin basis under the action of the algebra generators are written down. An analogoue of the Poincaré-Birkhoff-Witt theorem is formulated.

1. Introduction

During the last years the quantum groups became a field of increasing interest in various branches of physics and mathematics. The concept of a quantum group was introduced by Drinfeld [6]. Its essence crystallized from the intensive development of the quantum inverse problem method [8] and the investigations related to the Yang-Baxter equation (see the collection of papers [14] and the references therein).

An important class of quantum groups are the quantized universal enveloping algebras, called also quantum algebras. A quantum algebra $U_q[G]$ associated with the algebra G is a deformation of the universal enveloping algebra U[G] of G endowed with a structure of a Hopf algebra. In all applications we know these are one-parameter deformations (see, however, [28]). The first example of a Hopf algebra of this kind was given for G = sl(2) [23]. The generalization to any Kac-Moody Lie algebra with a symmetrizable generalized Cartan matrix is due to Drinfeld [7] and Jimbo [15]. An example of a quantum superalgebra, i.e., of a quantum algebra (LS) osp(1/2) was considered by Kulish [24]. The corresponding construction for an arbitrary Kac-Moody superalgebra with a symmetrizable generalized Cartan matrix and the symmetrizable generalized Cartan matrix was reported in [39]; an independent approach for the basic Lie superalgebras [18] was developed in [2].

^{*} Present address