Commun. Math. Phys. 141, 523–531 (1991)

Communications in
Mathematical
Physics

How to Find the Lax Pair from the Yang–Baxter Equation*

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Received October 25, 1990; in revised form February 25, 1991

Abstract. We show explicitly how to construct the quantum Lax pair from the Yang–Baxter equation. We demonstrate the new method by applying it to the Heisenberg XYZ model.

1. Introduction

It is well-known that the Yang–Baxter equations (YBE) play a crucial role in classical and quantum integrable systems (see e.g. [1, 2]). The structure and utility of the classical YBE is now fairly well understood [3, 4]. Unfortunately, it is much less so for its quantum partner. Although fruitful interactions between $(1+1)$-dimensional quantum field theory and $2d$ classical mechanics have led Faddeev, Sklyanin and Takhtajan to propose the quantum inverse scattering method (QISM) [5, 6] as a unified extension of the classical integrable models (soliton theory) to the quantum case, the exact relation between the R-matrix and the Lax pair [7] is still not clear.

Consider an operator version of an auxiliary linear problem [8],

$$\Psi_{n+1} = L_n(u)\Psi_n, \quad \dot{\Psi}_n = P_n \Psi_n, \tag{1}$$

where $L_n(u)$ and P_n are matrix operators, u is the spectral parameter and a dot signifies a time derivative. The consistency condition for (1) with $\dot{u} = 0$ yields the Lax pair equation

$$\dot{L}_n = P_{n+1}L_n - L_n P_n. \tag{2}$$

All the solved integrable models appear to imply that "a model is completely integrable if we can find a Lax pair $\{L_n, P_n\}$ such that the Lax equation (2) is equivalent to the equation of motion of the model" [8]. One may raise a serious

* Supported by DOE contract DE-FG02-88ER25053