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Fourier Analysis on a Hyperbolic Supermanifold with Constant Curvature

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Abstract. The Fourier inversion theorem is proved for a rank-one noncompact homogeneous space, the hyperbolic superplane. The proof makes use of some novel features of perfectly graded superspaces, which are not encountered in classical geometric analysis. An application to quasi-one-dimensional disordered one-electron systems is given.

0. Introduction

The theory of quantum transport and localization in disordered one-electron systems distinguishes between three universality classes, each being described by a statistical ensemble of Hamilton operators with local gauge invariance, which is either orthogonal, or unitary, or symplectic. Avoiding the replica trick used in the pioneering work of Wegner [15] and of Wegner and Schäfer [11], Efetov [5] showed how to calculate ensemble averages of products of Green's functions for each universality class, by means of a mapping onto nonlinear σ models with super coset spaces $\mathbf{G}/\mathbf{K}$ for their target spaces. A fruitful advance in extracting physical information from these models has recently been made by Iida, Weidenmüller and Zuk [8]. Starting from the Landauer–Büttiker formula for the conductance [12], and using S-matrix techniques developed in the context of statistical nuclear reaction theory [9, 13], they mapped the problem of calculating the average conductance of a quasi-one-dimensional metallic system onto the problem described in the next paragraph.

With e being the unit element of $\mathbf{G}$, let $o = e\mathbf{K}$ denote the origin of the space $\mathbf{G}/\mathbf{K}$ that is associated with the disordered conductor under consideration. The elements of $\mathbf{G}/\mathbf{K}$ are left cosets $g\mathbf{K}$ ($g \in \mathbf{G}$), which are written $g \cdot o \overset{\text{def}}{=} g\mathbf{K}$, the operator "$\cdot$" denoting the transitive action of $\mathbf{G}$ on $\mathbf{G}/\mathbf{K}$ by left translation. Let $\mathbf{G}/\mathbf{K}$ be given its natural $\mathbf{G}$-invariant geometry, let $\Delta_{\mathbf{G}/\mathbf{K}}$ denote the Laplace–Beltrami operator on $\mathbf{G}/\mathbf{K}$, and let $f(g \cdot o; s)$ be the solution of the heat equation

$$\partial_s f = \Delta_{\mathbf{G}/\mathbf{K}} f \tag{0.1}$$