

# Classification of Irreducible Super-Unitary Representations of $\mathfrak{su}(p, q/n)$

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**Abstract.** In this paper we classify all the irreducible super-unitary representations of  $\mathfrak{su}(p, q/n)$ , which can be integrated up to a unitary representation of  $S(U(p, q) \times U(n))$ , a Lie group corresponding to the even part of  $\mathfrak{su}(p, q/n)$ . Note that a real form of the Lie superalgebra  $\mathfrak{sl}(m/n; \mathbb{C})$  which has non-trivial super-unitary representations is of the form  $\mathfrak{su}(p, q/n)(p + q = m)$  or  $\mathfrak{su}(m/r, s)(r + s = n)$ . Moreover, we give an explicit realization for each irreducible super-unitary representation, using the oscillator representation of an orthosymplectic Lie superalgebra.

## Introduction

The theory of Lie superalgebras and their representations have come to play an important role in physics in recent years. They appear in several fields of physics such as elementary particle physics, nuclear physics, theory of supergravity and so on (cf. [20]).

Much fundamental work regarding basic classical Lie superalgebras and their finite dimensional representations has been produced by V. G. Kac ([16–18]), who classified all the finite dimensional simple Lie superalgebras. Thereafter, in mathematics, many interesting papers on these algebras and their representations have appeared.

In the early stages, mainly finite dimensional representations were studied. Irreducible representations of simple Lie superalgebras are divided into typical and atypical ones according to their central character ([18]). Finite-dimensional typical irreducible representations have many properties in common with the finite dimensional irreducible representations of simple Lie algebras ([1, Chap. II.5; 16, 18]). But atypical representations are not so easy to treat even if they are finite dimensional. Properties of atypical (finite dimensional) representations have not been studied sufficiently (cf. [7]).