Commun. Math. Phys. 141, 423–440 (1991)

Communications in Mathematical Physics

Perturbative Expansion of Chern-Simons Theory with Non-Compact Gauge Group*

Dror Bar-Natan[1,**] **and Edward Witten**[2]

[1] Department of Mathematics, Princeton University, Princeton, NJ 08544, USA
[2] School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, NJ 08540, USA

Received April 1, 1991

Abstract. Naive imitation of the usual formulas for compact gauge group in quantizing three dimensional Chern-Simons gauge theory with non-compact gauge group leads to formulas that are wrong or unilluminating. In this paper, an appropriate modification is described, which puts the perturbative expansion in a standard manifestly "unitary" format. The one loop contributions (which differ from naive extrapolation from the case of compact gauge group) are computed, and their topological invariance is verified.

1. Introduction

In evaluating Feynman diagrams in gauge theories, one encounters the Casimir invariants of the gauge group G and of whatever matter representations may be present. In conventional Yang-Mills theory with the usual F^2 action, the Feynman diagrams depend on G only through the values of these Casimirs. One might expect that the same would be true in three dimensional gauge theory with the pure Chern-Simons action. We consider a G bundle E, with connection A, over an oriented manifold M. The Chern-Simons functional is[1]

$$I(A)=\frac{1}{4\pi}\int_M \mathrm{Tr}\left(A\wedge dA+\frac{2}{3}A\wedge A\wedge A\right) \tag{1.1}$$

and the Lagrangian is

$$L=-ikI(A) \tag{1.2}$$

* Research supported in part by NSF Grant PHY86-20266

** Present address: Department of Mathematics, Harvard University, Cambridge, MA 02138, USA

[1] Here for $G=SU(N)$, or any real form thereof. Tr is the trace in the N dimensional. In general, for simply connected G, Tr is the smallest multiple of the trace in the adjoint representation such that $I(A)$ is well defined with values in $\mathbb{R}/2\pi\mathbb{Z}$. If G is not simply connected, we use the same definition of Tr as for the simply connected cover. The generators of a compact Lie group are skew symmetric matrices, so the quadratic form $(a,b)=-\mathrm{Tr}\,ab$ is positive definite for compact G