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Fusion Rings and Geometry*

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Abstract. The algebraic structure of fusion rings in rational conformal field theories is analyzed in detail in this paper. A formalism which closely parallels classical tools in the study of the cohomology of homogeneous spaces is developed for fusion rings, in general, and for current algebra theories, in particular. It is shown that fusion rings lead to a natural orthogonal polynomial structure. The rings are expressed through generators and relations. The relations are then derived from some potentials leading to an identification of the fusion rings with deformations of affine varieties. In general, the fusion algebras are mapped to affine varieties which are the locus of the relations. The connection with modular transformations is investigated in this picture. It is explained how chiral algebras, arising in $N = 2$ superconformal field theory, can be derived from fusion rings. In particular, it is argued that theories of the type $SU(N)_k/SU(N-1)$ are the $N = 2$ counterparts of Grassmann manifolds and that there is a natural identification of the chiral fields with Schubert varieties, which is a graded algebra isomorphism.

1. Introduction

In recent years much interest has been focused on conformal field theory in two dimensions in connection with string theory and two dimensional critical phenomena. It was argued that string theories in four dimensions are described by some conformal field theories and that the properties of the string theories are a consequence of the characteristics of the two dimensional theory. A particularly fruitful set of ideas is the relation between algebraic properties of the conformal field theory and the emergence of space-time geometry. It was demonstrated, and conjectured to hold in general [1], that all $N = 2$ superconformal field theories are in one to one relation with complex manifolds. The interplay between algebraic

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