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Scalings in Circle Maps II

F. M. Tangerman[1] and J. J. P. Veerman[2]

[1] Mathematics Department, SUNY at Stony Brook, Stony Brook, NY 11794, USA
[2] Institute for the Mathematical Sciences, SUNY at Stony Brook, Stony Brook, NY 11794, USA

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Abstract. In this paper we consider one parameter families of circle maps with nonlinear flat spot singularities. Such circle maps were studied in [Circles I] where in particular we studied the geometry of closest returns to the critical interval for irrational rotation numbers of constant type. In this paper we apply those results to obtain exact relations between scalings in the parameter space to dynamical scalings near parameter values where the rotation number is the golden mean. Then results on [Circles I] can be used to compute the scalings in the parameter space. As far as we are aware, this constitutes the first case in which parameter scalings can be rigorously computed in the presence of highly nonlinear (and nonhyperbolic) dynamics.

0. Introduction

In this paper we consider one parameter families of circle maps with nonlinear flat spot singularities. Such circle maps were studied in [Circles I] where in particular we investigated the geometry of closest returns to the critical interval for irrational rotation numbers of constant type. In this paper we apply those results to relate scalings in the parameter space to dynamical scalings near parameter values where the rotation number has constant type. That one should be able to establish such a relation is part of the renormalization philosophy for one-dimensional dynamical systems. The case we study here constitutes the first example in the presence of nonlinear singularities where such relations can be rigorously established. We restrict ourselves to the case where the rotation number is the golden mean.

Let f_0 be a circle map with a flat spot singularity which has bounded nonlinearity on the left side of the singular interval and has a power-law ($x \to x^{\nu_r}$) singularity on the right side. Assume that f_0 has golden mean rotation number. Let f_t be a one parameter family of such maps such that $\left.\frac{d}{dt}\right|_{t=0} f_t(x)$ is nonvanishing. Such families occur naturally as truncations of families of smooth bimodal maps.