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2+1 Gravity for Genus >1

J. E. Nelson and T. Regge

Dipartimento di Fisica Teorica dell'Università di Torino, Via Pietro Giuria 1, I-10125 Torino, Italy

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Abstract. In [1] we analysed the algebra of observables for the simple case of a genus 1 initial data surface Σ^2 for 2+1 De Sitter gravity. Here we extend the analysis to higher genus. We construct for genus 2 the group of automorphisms H of the homotopy group π_1 induced by the mapping class group. The group H induces a group D of canonical transformations on the algebra of observables which is related to the braid group for 6 threads.

1. Introduction

In [1, 2] we have derived the algebra of observables for quantum gravity in 2+1 dimensions, when the spatial hypersurfaces are genus 1 Riemann surfaces [3], namely tori. The cases without [1] and with [2] a cosmological constant were discussed, and in the case of the De Sitter theory, it was shown that the quantum algebra of gauge invariant quantities, i.e. observables, is trivially related to the quantum group $SU(2)_q$ [4].

In this paper the analysis is extended to higher genus. The classical algebra of observables is explicitly defined and calculated for genus $g=2$. There are at least two isomorphic and independent sets of observables with corresponding isomorphic symplectic structures. Identities satisfied by traces of $SL(2, R)$ matrices used in the representations of the fundamental group $\pi_1(\Sigma^2, B)$, where B is the base point on the initial data surface Σ^2, are used systematically and pose no problem at the classical level. Similarly an additional set of identities follows from the relator of π_1 which fixes the genus to be exactly g. It is not yet clear which role these combined sets of identities should play at the quantum level.

In Sect. 2 we review notations and conventions and set the stage for the calculation of the algebra A of observables for genus $g \geqq 2$. In Sect. 3 this algebra is discussed in detail. In Sect. 4 we discuss the role and the relationships among the mapping-class group, the braid group and their representations in terms of canonical transformations on A. Relevant formulas are presented in the Appendix.