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A Ruelle Operator for a Real Julia Set

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Abstract. Let R be an expanding rational function with a real bounded Julia set, and let $(Lg)(x)=\sum_{Ry=x}\frac{g(y)}{[R'(y)]^2}$ be a Ruelle operator acting in a space of functions analytic in a neighbourhood of the Julia set. We obtain explicit expressions for the resolvent function $E(x,z;\lambda)=(I-\lambda L)^{-1}\frac{1}{z-x}$ and, in particular, for the Fredholm determinant $D(\lambda)=\det(I-\lambda L)$. It gives us an equation for calculating the escape rate. We relate our results to orthogonal polynomials with respect to the balanced measure of R. Two examples are considered.

1. Introduction

The facts from the Fatou-Julia theory of iterations used below are contained, for example, in the surveys of Blanchard [6], and Milnor [15]. We shall use also some notions of the thermodynamic formalism for expanding mappings developed in the works of Sinai, Ruelle and Bowen (e.g. see Bowen [7, Chap. 1, 2], and the recent survey of Ruelle [18], which is supplied with an extensive list of references).

Let R be a rational function with a real bounded Julia set J. We shall assume that the mapping R is expanding on J (another word: hyperbolic), that is, for some $A>0$, $c>1$, and all integers $n>0$,

$$\inf\{|R_n'(x)| : x\in J\}\geqq Ac^n, \tag{1.1}$$

where R_n is the n^{th} iteration of R [in the case of real bounded Julia set the inequality (1.1) is equivalent to the conditions: R has not neutral fixed points and critical points on J, see Sect. 2.1]. Under these hypotheses J is a Cantor-type set of zero length.

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