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Entropy Production by Block Variable Summation and Central Limit Theorems

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Abstract. We prove a strict lower bound on the entropy produced when independent random variables are summed and rescaled. Using this, we develop an approach to central limit theorems from a dynamical point of view in which the entropy is a Lyapunov functional governing approach to the Gaussian limit. This dynamical approach naturally extends to cover dependent variables, and leads to new results in pure probability theory as well as in statistical mechanics. It also provides a unified framework within which many previous results are easily derived.

Introduction

We develop an approach to central limit theorems for dependent as well as independent random variables based on ideas from dynamical systems and information theory, and we obtain new results in statistical mechanics and probability theory using it. The approach is motivated in part by Jona-Lasinio's study of the relation between renormalization group methods in statistical mechanics and the classical limit theorems of probability [JoL], and in part by Linnik's information-theoretic proof of the central limit theorem [Lin59]. In renormalization group analysis, one considers block summation of random variables as an iterative dynamics on the state space of a dynamical system whose states are certain arrays of random variables [Gaw]. What we show here is that entropy often serves as a good Lyapunov functional for the convergence to a Gaussian fixed point under this dynamics. This results in a clean implementation of Linnik's ideas in a broad setting with significant additional consequences.

Our results are based on a detailed study of entropy production in the process of summing independent random variables; or equivalently, the entropy produc-

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