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Quantum Yang–Mills on a Riemann Surface

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Abstract. We obtain the quantum expectations of gauge-invariant functions of the connection on a $G = SU(N)$ product bundle over a Riemann surface of genus g. We show that the space $\mathscr{A}/\mathscr{G}_m$ of connections modulo those gauge transformations which are the identity at one point is itself a principal bundle with affine linear fiber. The base space $\mathrm{Path}^{2g} G$ consists of $2g$-tuples of paths in G subject to a relation on their endpoint values. Quantum expectations are iterated path integrals over first the fiber then over $\mathrm{Path}^{2g} G$, each with respect to the push-forward to $\mathscr{A}/\mathscr{G}_m$ of the measure $e^{-S(A)}\mathscr{D}A$. Here, $S(A)$ denotes the Yang–Mills action on $\mathscr{A}$. We exhibit a global section of $\mathscr{A}/\mathscr{G}_m$ to define a choice of origin in each fiber, relative to which the measure on the fiber is Gaussian. The induced measure on $\mathrm{Path}^{2g} G$ is the product of Wiener measures on the component paths, conditioned to preserve the endpoint relation. Conformal transformations of the metric on M act by reparametrizing these paths. We explicitly compute the partition function in the general case and the expectations of functions of certain products of Wilson loops in the case $g = 1$.

Introduction

In [2], we treated Yang–Mills on S^2, deriving the quantum expectation of a gauge-invariant function of the connection. To do so, we interpreted the path intergal as an integral with respect to a measure μ on $\mathscr{A}/\mathscr{G}_m$, the space of connections modulo gauge transformations which are the identity at a point m. We showed that $\mathscr{A}/\mathscr{G}_m$ fibers over ΩG, based loops in the symmetry group, and we formally decomposed μ into a measure on the fiber and a measure on the base.

Sengupta [5] treats the same problem from the perspective of stochastic parallel transports, as developed for Yang–Mills on R^2 in Gross, King and Sengupta [4]. His results and those of [2] agree where they overlap. In a future paper, we intend to check for further agreement.

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