Commun. Math. Phys. 140, 43–62 (1991)

Communications in
Mathematical
Physics

Absence of Highest-Spin Ground States in the Hubbard Model

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Received July 23, 1990

Abstract. The Hubbard model $H = -t\,\Sigma c^{\dagger}_{x\sigma} c_{y\sigma} + U\,\Sigma n_{x\uparrow} n_{x\downarrow}$ with N electrons and periodic boundary condition is studied on ν-dimensional $L_1 \times \cdots \times L_\nu$ lattices. It is shown that for any value of U there is no ground state with maximal spin ($S = N/2$) in the following cases: (*i*) $\mathbb{Z}^\nu$ ($\nu \geqq 2$) at low electron densities; with one hole if $t > 0$ and L_i is odd for some *i*; with two holes if $t < 0$, or if $t > 0$ and all the L_i are even. (ii) The *bcc* lattice at low densities; with two holes if $t < 0$, or if $t > 0$ and all the L_i are even; with $2, \ldots, 6$ holes if $L_i = L$ and $t < 0$, or if $t > 0$ and L is even. (iii) The triangular lattice at densities near 0 and 1 if $t > 0$; with two holes if $t < 0$; with 2, 3, 4 holes if $t < 0$ and $L_1 = L_2$. (iv) The *fcc* lattice at densities near 0 and 1 if $t > 0$; with two holes if $t < 0$. Some results for the one dimensional model are also presented.

1. Introduction

To prove the appearance of ferromagnetism in some reasonable model of itinerant electrons has long been the obsession of theoretical and mathematical physicists. Perhaps the simplest such model is the Hubbard model. On a finite lattice Λ it is given by the Hamiltonian

$$H_U = -\sum_{x \neq y \in \Lambda} t_{xy}(c^{\dagger}_{x\uparrow} c_{y\uparrow} + c^{\dagger}_{x\downarrow} c_{y\downarrow}) + U \sum_{x \in \Lambda} n_{x\uparrow} n_{x\downarrow} = H_0 + U H_I. \tag{1.1}$$

Here $c_{x\sigma}$ and its adjoint $c^{\dagger}_{x\sigma}$ are fermion annihilation and creation operators which satisfy the anticommutation relations

$$\begin{aligned} c^{\dagger}_{x\sigma} c_{y\tau} + c_{y\tau} c^{\dagger}_{x\sigma} &= \delta_{xy}\delta_{\sigma\tau}, \\ c_{x\sigma} c_{y\tau} + c_{y\tau} c_{x\sigma} &= 0. \end{aligned} \tag{1.2}$$

* On leave from the Central Research Institute for Physics, Budapest