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Long Range Scattering for Nonlinear Schrödinger Equations in One Space Dimension

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Abstract. We consider the scattering problem for the nonlinear Schrödinger equation in 1 + 1 dimensions:

$$i\partial_t u + (1/2)\partial^2 u = \lambda |u|^2 u + \mu |u|^{p-1} u, \quad (t,x) \in \mathbb{R} \times \mathbb{R}, \tag{$*$}$$

where $\partial = \partial/\partial x$, $\lambda \in \mathbb{R} \setminus \{0\}$, $\mu \in \mathbb{R}$, $p > 3$. We show that modified wave operators for $(*)$ exist on a dense set of a neighborhood of zero in the Lebesgue space $L^2(\mathbb{R})$ or in the Sobolev space $H^1(\mathbb{R})$. The modified wave operators are introduced in order to control the long range nonlinearity $\lambda |u|^2 u$.

1. Introduction

In this paper we consider the asymptotic behavior in time of solutions to the Schrödinger equations with power nonlinearities:

$$i\partial_t u + (1/2)\partial^2 u = f(u), \quad (t,x) \in \mathbb{R} \times \mathbb{R}, \tag{1.1}$$

where u is a complex valued function on $\mathbb{R} \times \mathbb{R}$, $\partial_t = \partial/\partial t$, $\partial = \partial/\partial x$, and f is a complex valued function on $\mathbb{C}$. A typical form of $f(u)$ is the sum of two powers

$$f(u) = \lambda |u|^{q-1} u + \mu |u|^{p-1} u \tag{1.2}$$

with $p \geqq q \geqq 1$, $\lambda, \mu \in \mathbb{R}$.

There is a large literature on the equations of the form (1.1) from both mathematical and physical point of view, see [1–4, 7–17, 19–26, 28–30]. Let $H^{m,s}$ be the weighted Sobolev space defined by

$$H^{m,s} = \{\psi \in \mathscr{S}'; \|\psi\|_{m,s} = \|(1 + |x|^2)^{s/2}(1 - \partial^2)^{m/2}\psi\|_2 < \infty\}, \quad m, s \in \mathbb{R},$$

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