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Periodic and Partially Periodic Representations of $SU(N)_q$

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Abstract. The Gelfand–Zetlin basis is adapted to $SU(N)_q$ for q a root of unit. Extra parameters are incorporated in the matrix elements of the generators to obtain all the invariants corresponding to the augmented center. A crucial identity is derived and proved, which guarantees the periodicity of the action of the generators. Full periodicity is relaxed by stages, some raising and lowering operators remaining injective while others become nilpotent with corresponding changes in the dimension of the representation. In the extreme case of highest weight representations, all the raising and lowering operators are nilpotent. As an alternative approach an auxiliary algebra giving all the periodic representations is presented. An explicit solution of this system for $N=3$, while fully equivalent to the G.–Z. basis, turns out to be much simpler.

1. Introduction

The Gelfand–Zetlin basis was proposed [1] for the classical ($q = 1$) $U(N)$ and $O(N)$ groups. This was extended [2] to non-semisimple $IU(N)$ and $IO(N)$ groups, which have an Abelian subalgebra in semidirect product with the homogeneous $U(N)$ and $O(N)$ subalgebras respectively. The q-analogues of both the cases, homogeneous [3] and inhomogeneous [4] were then proposed for the unitary case and for q not a root of unity. It will be shown in Sect. 2 that the G.–Z. basis works also for q a root of unity if the domains of the parameters involved are chosen suitably. In particular periodicity requirements can be imposed systematically. For $SU(2)$ and $SU(3)$ periodic representations [5, 6] were classified elsewhere. Their relations with the corresponding representation in the G.–Z. basis will be given. But the G.–Z. basis, adapted to the root of unity case works canonically for any N. One can impose full periodicity in all the parameters. As proved in Sect. 3, one can also relax these constraints by stages. To each stage corresponds a typical domain of the parameters involved. Study of such constraints is rewarding. They probe and display fully the possibilities of the formalism. An alternative study is developed