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Higher-Order Polarizations on the Virasoro Group and Anomalies*

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Abstract. In a previous paper the authors showed that the space of (first order) polarized functions on the Virasoro group is not, in general, irreducible. The full reduction was explicitly achieved by taking the orbit of the enveloping algebra through the vacuum. This additional step provided the proper quantization in the "strong-coupling" domain $0 < c \leqq 1$. In this paper we introduce the concept of "higher order polarization" as a generalization of that of polarization. We prove that the imposing of the additional (higher-order) polarization conditions is equivalent to the taking of the above-mentioned orbit. This demonstrates that the generalized (higher-order) polarization conditions suffice to obtain the irreducible Hilbert spaces. We also discuss the need for higher order polarizations in terms of anomalies.

1. Introduction

The standard geometric methods for quantizing a phase space (symplectic manifold) make use of the concept of polarization. Generally speaking, given a symplectic manifold (M, ω) the corresponding quantum Hilbert space is the space of sections of a line bundle (whose first Chern class is $[\omega]$) once the polarization conditions are imposed. A polarization P is an isotropic differential system ($\omega(X, X') = 0$ if $X', X \in P$) and the polarization conditions are

$$\nabla_X \psi = 0 \quad \text{if} \quad X \in P, \tag{1.1}$$

where ψ are sections of the line bundle and ∇ is a covariant derivative whose curvature is given by the symplectic form. The conditions (1.1) are always first-

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