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Interactions and Pressure Functionals for Disordered Lattice Systems

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Abstract. The purpose of this paper is to provide a theoretical framework for disordered spin systems on a lattice, similar to that of classical statistical mechanics in the sense of Ruelle [Ru]. We prove the existence of a continuous pressure functional on a large Banach space of random interactions (highly generalizing the classical one) and formulate an analog of the variational principle.

0. Introduction

The statistical mechanics of disordered lattice systems, considered as a mathematical description of models of real materials such as e.g. real crystals, alloys of magnetic and nonmagnetic metals, glasses and others, have to deal with new phenomena and problems. For general reviews and connection to other domains of physics, mathematics and biology see e.g. [4, 25, 36, 48, 11].

It was observed by Edwards and Anderson [15], that it is very useful to describe such systems as a lattice model with random interactions. The randomness and existence of competing interactions allow a system to behave thermodynamically well even if the decay of interactions is very slow and its amplitude is a'priori unbounded. The first rigorous proof of this fact has been given in [40, 33], where the existence of a (nonrandom) thermodynamic limit of the pressure has been proven for a random bond Ising model given by a hamiltonian function

$$H_0 \equiv -\sum_{i \neq j} J_{ij}\sigma_i\sigma_j; \qquad i,j \in \mathbb{Z}^d \tag{0.1}$$

with $\{J_{ij}\}$ being independent random variables satisfying for all $i,j \in \mathbb{Z}^d$, $i \neq j$,

$$EJ_{ij} = 0, \tag{0.2a}$$

and for all $n \in \mathbb{N}$

$$|EJ_{ij}^n| \leq n!\,\gamma^n |i-j|^{-n\alpha d} \tag{0.2b}$$

with some constants $0 < \gamma < \infty$ and $\frac{1}{2} < \alpha < \infty$.

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