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Discrete Versions of Some Classical Integrable Systems and Factorization of Matrix Polynomials

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Abstract. Discrete versions of several classical integrable systems are investigated, such as a discrete analogue of the higher dimensional force-free spinning top (Euler-Arnold equations), the Heisenberg chain with classical spins and a new discrete system on the Stiefel manifold. The integrability is shown with the help of a Lax-pair representation which is found via a factorization of certain matrix polynomials. The complete description of the dynamics is given in terms of Abelian functions; the flow becomes linear on a Prym variety corresponding to a spectral curve. The approach is also applied to the billiard problem in the interior of an N-dimensional ellipsoid.

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