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Classical Origin of Quantum Group Symmetries in Wess-Zumino-Witten Conformal Field Theory

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Abstract. We elucidate the way by which the quantum group symmetries of the WZW models arise within the canonical formalism of the classical field theory.

1. Introduction

The quantum group symmetries of the 2-dimensional field theories have attracted considerable attention recently. On one side, they lead to a better understanding of the structure of 2-dimensional integrable models and of their exact solutions. On the other side, they allow us to learn more about the quantum groups [11, 38, 52] themselves and about the non-commutative geometry underlying that concept. In particular, multiple relations to quantum groups were discovered in the study of the exchange algebra of chiral vertex operators of numerous conformal field theories like WZW models, minimal models, Liouville and Toda theories [42, 2, 3, 17, 4, 7] leading finally to the realization that quantum groups should describe the symmetries of the chiral sector of conformal field theories [43, 18, 29, 48, 35–37, 20, 30, 19].

At the very origin of quantum groups lay the fact that they were the integrated version of simpler, infinitesimal structures abstracted from the canonical formalism of classical integrable theories [12, 14]. Hence the interest in exhibiting the classical origin of quantum symmetries of conformal field theories. An important step in this direction was achieved in [5] in the context of the Liouville theory and was used in [29, 30] to explicit the quantum group symmetries of the quantized Liouville theory constructed earlier by Gervais-Neveu [31, 32]. In [13], Faddeev has adapted those observations to the case of WZW model of conformal fields and in [1] Alekseev-Shatashvili have provided still more geometric explanation for the appearance of quantum groups in the chiral WZW models.

The present paper may be viewed as a continuation of [13, 1]. It lays down a more systematic version of the canonical formalism first for the complete WZW model and then for its chiral part. Within this formalism, we show that the left-