Commun. Math. Phys. 139, 103–139 (1991)

Communications in Mathematical Physics

Boundary Conditions for Quantum Mechanics on Cones and Fields Around Cosmic Strings

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Received August 13, 1990; in revised form January 2, 1991

Abstract. We study the options for boundary conditions at the conical singularity for quantum mechanics on a two-dimensional cone with deficit angle $\leqq 2\pi$ and for classical and quantum scalar fields propagating with a translationally invariant dynamics in the $1+3$ dimensional spacetime around an idealized straight infinitely long, infinitesimally thin cosmic string. The key to our analysis is the observation that minus-the-Laplacian on a cone possesses a one-parameter family of self-adjoint extensions. These may be labeled by a parameter R with the dimensions of length – taking values in $[0, \infty)$. For $R=0$, the extension is positive. When $R \neq 0$ there is a bound state. Each of our problems has a range of possible dynamical evolutions corresponding to a range of allowed R-values. They correspond to either finite, for $R=0$, or logarithmically divergent, for $R \neq 0$, boundary conditions at zero radius. Non-zero R-values are a satisfactory replacement for the (mathematically ill-defined) notion of δ-function potentials at the cone's apex.

We discuss the relevance of the various idealized dynamics to quantum mechanics on a cone with a rounded-off centre and field theory around a "true" string of finite thickness. Provided one is interested in effects at sufficiently large length scales, the "true" dynamics will depend on the details of the interaction of the wave function with the cone's centre (/field with the string etc.) only through a single parameter R (its "scattering length") and will be well-approximated by the dynamics for the corresponding idealized problem with the same R-value. This turns out to be zero if the interaction with the centre is purely gravitational and minimally coupled, but non-zero values can be important to model non-gravitational (or non-minimally coupled) interactions. Especially, we point out the relevance of non-zero R-values to electromagnetic waves around superconducting strings. We also briefly speculate on the relevance of the R-parameter in the application of quantum mechanics on cones to $1+2$ dimensional quantum gravity with massive scalars.