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Microcanonical Distributions for Lattice Gases

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Abstract. In this article, a large deviation principle (cf. Theorem 1.3) for the empirical distribution functional is applied to prove a rather general version of Boltzmann's principle (cf. Theorem 3.5) for models with shift-invariant, finite range potentials. The final section contains an application of these considerations to the two dimensional Ising model at sub-critical temperature.

1. A Large Deviation Principle for Lattice Systems

In this section we will prove a large deviation theorem for families of random variables indexed by points on a square lattice. (Related earlier results in this direction can be found in [C, F0, and O].) Thus, let $\mathbb{Z}^d$ be the d-dimensional square lattice. We will write $\Lambda \subset\subset \mathbb{Z}^d$ if Λ is a non-empty finite subset of $\mathbb{Z}^d$ and use $|\Lambda| \in \mathbb{Z}^+$ to denote the cardinality of Λ. Also, for $R \in \mathbb{Z}^+$ and $\Lambda \subset\subset \mathbb{Z}^d$, we define

$$\Lambda(R) \equiv \{\mathbf{k} \in \mathbb{Z}^d : |\mathbf{k} - \Lambda| \leqq R\} \quad \text{and} \quad \partial_R \Lambda \equiv \Lambda(R) \backslash \Lambda$$

to be, respectively, the R-hull and R-boundary of Λ. (Throughout, $|\mathbf{k}| \equiv \max_{i \leqq i \leqq d} |k_i|$.)

Next, let E be a Polish space, $\mathscr{B}_E$ the Borel field over E, and $\Omega = E^{\mathbb{Z}^d}$. We give Ω the product topology, and use $\mathscr{B}_\Omega$ to denote the associated Borel field over Ω. Given a non-empty $\Lambda \subseteqq \mathbb{Z}^d$ and $\mathbf{x} \in \Omega$, $\mathbf{x}_\Lambda$ will denote the element of E^Λ obtained by restricting $\mathbf{x}$ to Λ, $\mathscr{B}_\Lambda$ is the σ-algebra over Ω generated by the projection map $\mathbf{x} \in \Omega \to \mathbf{x}_\Lambda \in E^\Lambda$ (of course, $\mathscr{B}_\Omega = \mathscr{B}_{\mathbb{Z}^d}$), $B_\Lambda(\Omega; \mathbb{R})$ is the set of bounded $\mathbb{R}$-valued, $\mathscr{B}_\Lambda$-measurable functions on Ω, and $C_{\Lambda,b}(\Omega; \mathbb{R})$ is the subset of continuous elements of $B_\Lambda(\Omega; \mathbb{R})$. When $\Lambda = \mathbb{Z}^d$, we will simply write $B(\Omega; \mathbb{R})$ for $B_{\mathbb{Z}^d}(\Omega; \mathbb{R})$ and $C_b(\Omega; \mathbb{R})$

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