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Conformal Field Theories via Hamiltonian Reduction

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Abstract. Constraining the $SL(3)$ WZW-model we construct a reduced theory which is invariant with respect to the new chiral algebra W_3^2. This symmetry is generated by the stress-energy tensor, two bosonic currents with spins 3/2 and the $U(1)$ current. We conjecture a Kac formula that describes the highly reducible representation for this algebra. We also discuss the quantum Hamiltonian reduction for the general type of constraints that leads to the new extended conformal algebras.

1. Introduction

It was recently observed [1, 2] there are hidden relations between the Virasoro algebra and the $SL(2, \mathscr{R})$ current algebra, and in general, between the W_n-algebra and the $SL(n, \mathscr{R})$ current algebra. The W_n-algebra is an extension of the Virasoro algebra with additional chiral operators of spin n [3, 4].

The relation between the current algebras and the extended conformal algebras is given by Drinfeld–Sokolov [5] reduction. One may regard the space dual to the loop algebra as the phase space endowed with a natural symplectic structure. This phase space possesses a certain symmetry and one may consider the reduced phase space under this symmetry. The Poisson Brackets on the reduced phase space are given by Drinfeld–Sokolov reduction [5] and coincide with the classical limit of extended conformal algebras.

Quantizing such theory one must replace the Poisson Brackets by commutation relations and classical phase space by an irreducible representation of the algebra. The irreducible representation spaces of the Virasoro algebra are extracted from those of the $SL(2, \mathscr{R})$ current algebra by imposing a certain constraint on the latter. Consider an irreducible representation space of the current algebra. In classical mechanics, we put a constraint $J^-(z) = 1$ to reduce the phase space of the loop algebra. Quantum mechanically, we introduce a set of ghosts and define the

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