Commun. Math. Phys. 138, 301–338 (1991)

Communications in Mathematical Physics

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Solving the Strongly Coupled 2D Gravity: 1. Unitary Truncation and Quantum Group Structure

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Received July 5, 1990; in revised form November 17, 1990

Abstract. Strongly compled gravity theories with Virasoro central charges equal to 7, 13, and 19 are shown to enjoy striking properties: at these values, the subset of chiral operators with real Virasoro-weights, acting on a subspace $\mathscr{H}_{\text{phys}}$, is shown to be closed by fusion and braiding, and to leave this subspace invariant. Moreover, the representation of the Virasoro algebra becomes unitary when it is restricted to $\mathscr{H}_{\text{phys}}$. Strongly coupled 2D gravity with $C_{\text{grav}}=7$, 13, or 19 may thus be naturally truncated obtaining a consistent conformal theory (this result is similar to the truncation that occurs for $C=1-6(p-p')^2/pp'$ with p and p' integers, where only a finite number of primary fields remains, as is well known in rational theories). The proof of this unitary truncation theorem, already summarized in a recent letter, is fully described here.

1. Introduction

This article is the third of a series [1–3] devoted to the solution of 2D gravity and minimal models by means of quantum groups. The present approach is a direct outcome of the algebraic approach to 2D critical systems which Neveu and I [4–6] introduced long ago. Its distinctive feature is that it directly deals with chiral operators that transform irreducibly under the action of the underlying quantum group, while in the more widespread type of approach [7], one works with Green functions and the link with quantum group is made at the level of group invariants and q–Clebsch Gordan or q-6-j symbols. Thus, in [7], one does not clearly see how the quantum group acts on the theory. Moreover, these approaches exclusively deal with rational theories at $C<1$. This is inappropriate for the strongly coupled gravity which we have in mind in the present paper. The special values $C_{\text{grav}}=7$, 13,

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