Commun. Math. Phys. 138, 51–61 (1991)

Communications in Mathematical Physics

Multiple-Mode Diffusion Waves for Viscous Nonstrictly Hyperbolic Conservation Laws*

I-Liang Chern

Mathematics and Computer Science Division, Argonne National Laboratory, Argonne, IL 60439-4801, USA and
Department of Mathematics, The University of Chicago, Chicago, IL 60637, USA

Received February 13, 1990; in revised form November 13, 1990

Abstract. We study the large-time behaviors of solutions of viscous conservation laws whose inviscid part is a nonstrictly hyperbolic system. The initial data considered here is a perturbation of a constant state. It is shown that the solutions converge to single-mode diffusion waves in directions of strictly hyperbolic fields, and to multiple-mode diffusion waves in directions of nonstrictly hyperbolic fields. The multiple-mode diffusion waves, which are the new elements here, are the self-similar solutions of the viscous conservation laws projected to the nonstrictly hyperbolic fields, with the nonlinear fluxes replaced by their quadratic parts. The convergence rate to these diffusion waves is $O(t^{-3/4+1/2p+\sigma})$ in L^p, $1 \leqq p \leqq \infty$, with $\sigma > 0$ being arbitrarily small.

1. Introduction

We are interested in the large-time behaviors of solutions of the viscous conservation laws

$$u_t + f(u)_x = u_{xx}, \quad u \in R^n, -\infty < x < \infty, t > 0, \tag{1.1}$$

whose inviscid part is a nonstrictly hyperbolic system. Physical models of the nonstrictly hyperbolic systems include, for instance, three-phase flows in oil reservoir [10]. The initial data considered here is a perturbation of a constant state. Without loss of generality, we may assume this constant state to be the zero state

$$u(x, 0) = u_0(x), \; u_0(x) \to 0, \quad \text{as} \quad |x| \to \infty. \tag{1.2}$$

The viscous term u_{xx} considered here is an idealized situation. The real physics has a more general viscous term $(B(u)u_x)_x$, where B is an $n \times n$ matrix. Kawashima showed that the nondiagonal part of $(B(u)u_x)_x$ decays faster than the diagonal

* This work was supported by the Applied Mathematical Sciences subprogram of the Office of Energy Research, U.S. Department of Energy, under Contract W-31-109-Eng-38