Commun. Math. Phys. 137, 617–639 (1991)

Communications in Mathematical Physics

Semi-Infinite Weil Complex and the Virasoro Algebra

Boris Feigin[1] **and Edward Frenkel**[2]

[1] Landau Institute for Theoretical Physics, Moscow, USSR
[2] Department of Mathematics, Harvard University, Cambridge, MA 02138, USA

Received October 10, 1990

Abstract. We define a semi-infinite analogue of the Weil algebra associated an infinite-dimensional Lie algebra. It can be used for the definition of semi-infinite characteristic classes by analogy with the Chern-Weil construction. The second term of a spectral sequence of this Weil complex consists of the semi-infinite cohomology of the Lie algebra with coefficients in its "adjoint semi-infinite symmetric powers." We compute this cohomology for the Virasoro algebra. This is just the BRST cohomology of the bosonic $\beta\gamma$-system with central charge 26. We give a complete description of the Fock representations of this bosonic system as modules over the Virasoro algebra, using Friedan-Martinec-Shenker bosonization. We derive a combinatorial identity from this result.

1. Introduction

It is well-known that the Weil algebra, associated to a finite-dimensional Lie algebra is very useful in geometry and topology.

Let us recall its definition.

Let G be a finite-dimensional Lie group, g – its Lie algebra. Denote by $\wedge^*(g')$ and $S^*(g')$ exterior and symmetric algebras of the dual space to g, correspondingly. Put $W(g)=\bigwedge^*(g')\otimes S^*(g')$. Introduce grading on $W(g)$:

$$W(g)=\bigoplus_{k\geqq 0} W^k(g),$$

where

$$W^k(g)=\bigoplus_{r+2p=k} \bigwedge^r(g')\otimes S^p(g').$$

Let X_i be basic elements of g. Denote by c_i and γ_i the images of X_i' in $\wedge^*(g')$ and $S^*(g')$, correspondingly. They are the generators of $W(g)$. Define the differential d in