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The Existence of Chern-Simons Vortices

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Abstract. A new type of vortices called Chern-Simons vortices is considered and the existence is established.

1. Introduction

Various vortex theories have been discussed in recent years, among them the classical, electrically charged and other ones. The discussions on vortices have proved important in quantum physics, solid state physics as well as in mathematics. A detailed account on classical (kinetic) vortex theory can be found in [14] by Jaffe and Taubes. When the Chern-Simons term is added to the action, this is $(2+1)$-dimensional massive electrodynamics, the vortex fields still exist and carry a fractional electric charge proportional to the coefficient of the Chern-Simons term. Such particles are called anyons, see Fröhlich and Marchetti [6].

The paper concerns the existence of Chern-Simons vortices recently found by physicists Jackiw and Weinberg, also Hong, Kim, and Pac who consider the problem of charged vortices with the gauge field governed solely by the Chern-Simons term. This truncation is physically sensible at large distances and low energies, where the Chern-Simons term dominates the higher-derivative Maxwell term. With the symmetry breaking realization, it is interesting that with a special choice of the Higgs potential the vortex solutions satisfy a Bogomolny type "self-dual" equation. We call the solutions Chern-Simons vortices, see [10] also [18].

Chern-Simons vortex solutions can be compared to the classical vortex solutions and anyons. Chern-Simons vortices satisfy a similar "self-dual" equation as classical vortices but the equation contains a more complicated nonlinear term. The existence of axial solutions to the equation have been given in [10] and with a special choice of metric on the plane the equation is completely solvable [16]. In this paper, along the course of [14], a complex analytical and variational method is used to establish the general existence of solutions to the Chern-Simons vortex equation. Denote by C or R^2 the standard 2-plane.