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# Conformal Embeddings, Rank-Level Duality and Exceptional Modular Invariants

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**Abstract.** We compute the branching rules of the conformal embeddings $SO(4nk)_1 \supset Sp(2n)_k \oplus Sp(2k)_n$ and $SO(rq)_1 \supset SO(r)_q \oplus SO(q)_r$ for $rq$ even. Using this we prove that the affine algebras $Sp(2n)_k$ and $Sp(2k)_n$ have the same $S$ matrix and modular invariants. As a second application, we show how the triality of $SO(8)$ leads to an exceptional modular invariant for $SU(2)$ at level 16 and for all $SO(q \geqq 4)$ at level 8.

## 1. Introduction

An important class of conformal field theories (CFT) [4] is provided by the Wess-Zumino-Witten (WZW) models [35]. They are characterized by the presence of a Kac-Moody (KM) symmetry [19, 17]. The WZW models lead to many other CFT's through the Goddard-Kent-Olive coset construction [16]. Examples are the minimal unitary Virasoro models [10] and the $N=2$ superconformal theories [23]. The latter have been used as building blocks for the internal sector of 4-dimensional heterotic strings [13, 23, 9]. The classification of modular invariant partition functions for Kac-Moody algebras is thus also important for string phenomenology.

Several authors have recently pointed out intriguing relations between a priori rather different KM algebras. Kac and Wakimoto [22] showed that $Sp(2n)_1$ and $SU(2)_n (= Sp(2)_n)$ have the same modular $S$ matrix (the subscript is the level; all the algebras considered here are untwisted). Walton [34] computed the branching rules of the conformal embeddings $SU(n)_k \oplus SU(k)_n \subset SU(nk)_1$ and observed that the modular invariant partition functions of $SU(n)_k$ are naturally related to those of $SU(k)_n$. Considering the same conformal embeddings, Altschuler, Bauer, and Itzykson [1] expressed the $S$ matrix of $SU(n)_k$ in terms of that of $SU(k)_n$. These embeddings were systematically used in [33] to obtain new exceptional modular

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