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The Geometry of the Super KP Flows

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Abstract. A supersymmetric generalization of the Krichever map is used to construct algebro-geometric solutions to the various super Kadomtsev-Petviashvili (SKP) hierarchies. The geometric data required consist of a suitable algebraic supercurve of genus g (generally *not* a super Riemann surface) with a distinguished point and local coordinates (z, θ) there, and a generic line bundle of degree $g-1$ with a local trivialization near the point. The resulting solutions to the Manin-Radul SKP system describe coupled deformations of the line bundle and the supercurve itself, in contrast to the ordinary KP system which deforms line bundles but not curves. Two new SKP systems are introduced: an integrable "Jacobian" system whose solutions describe genuine Jacobian flows, deforming the bundle but not the curve; and a nonintegrable "maximal" system describing independent deformations of bundle and curve. The Kac-van de Leur SKP system describes the same deformations as the maximal system, but in a different parametrization.

1. Introduction

The theory of the generalized KdV equations, or the KP hierarchy, stands at the crossroads of several flourishing branches of modern mathematics and physics: Riemann surfaces, algebraic geometry, integrable systems, loop groups, conformal field theory, string theory, and quantum gravity. The centerpiece of the theory is the construction of algebro-geometric solutions to this infinite system of nonlinear differential equations from geometric data, and the dual interpretation of the solutions as flows in the moduli space of geometric data or in an infinite-dimensional Grassmannian [1–6]. The geometric "Krichever" data consist of a Riemann surface with a choice of local coordinate near a distinguished point, and a generic line bundle of degree $g-1$ with a choice of local trivialization, and the flows deform the line bundle. The Krichever construction which produces the solutions has become a basic tool in the operator formalism of conformal field theory