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An Example of a Generalized Brownian Motion

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Abstract. We present an example of a generalized Brownian motion. It is given by creation and annihilation operators on a "twisted" Fock space of $L^2(\mathbb{R})$. These operators fulfill (for a fixed $-1 \leqq \mu \leqq 1$) the relations $c(f)c^*(g) - \mu c^*(g)c(f) = \langle f, g \rangle 1$ $(f, g \in L^2(\mathbb{R}))$. We show that the distribution of these operators with respect to the vacuum expectation is a generalized Gaussian distribution, in the sense that all moments can be calculated from the second moments with the help of a combinatorial formula. We also indicate that our Brownian motion is one component of an n-dimensional Brownian motion which is invariant under the quantum group $S_\nu U(n)$ of Woronowicz (with $\mu = \nu^2$).

1. Introduction

We will present a representation of the relations

$$c(f)c^*(g) - \mu c^*(g)c(f) = \langle f, g \rangle 1 \qquad (f, g \in L^2(\mathbb{R}))$$

for a fixed μ with $-1 \leqq \mu \leqq 1$ on a "twisted" Fock space (not to be confused with the twisted Fock space of Pusz and Woronowicz [PWo]). There are at least three reasons for studying these relations:

i) They provide an interpolation between the bosonic and fermionic relations. Independently from our work, Greenberg [Gre] proposed the same relations as a first (non-relativistic) field theory that allows small violations of the exclusion principle (i.e. of Fermi statistics) or of Bose statistics.

ii) They give an example of a generalized Brownian motion.

iii) They exhibit a relation with the twisted $S_\nu U(n)$ of Woronowicz [Wor 1, Wor 2]: the Brownian motion of ii) can be considered as one component of an n-dimensional Brownian motion which is $S_\nu U(n)$-invariant. This also shows how the twisted creation and annihilation operators of [PWo] (appearing in the second quantization procedure based upon the twisted $S_\nu U(n)$) arise in a central limit theorem.