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Duality and Modular Invariance in Rational Conformal Field Theories

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Abstract. We investigate the polynomial equations which should be satisfied by the duality data for a rational conformal field theory. We show that by these duality data we can construct some vector spaces which are isomorphic to the spaces of conformal blocks. One can construct explicitly the inner product for the former if one deals with a unitary theory. These vector spaces endowed with an inner product are the algebraic reminiscences of the Hilbert spaces in a Chern-Simons theory. As by-products, we show that the polynomial equations involving the modular transformations for the one-point blocks on the torus are not independent. We discuss the solution of structure constants for a physical theory. Making some assumption, we obtain a neat solution. And this solution in turn implies that the quantum groups of the left sector and of the right sector must be the same, although the chiral algebras need not be the same. Some examples are given. Finally, we discuss the reconstruction of the quantum group in a rational conformal theory.

1. Introduction

Despite the fact that much progress has been made in understanding the so-called rational conformal field theories (RCFT's) recently, there remain many open problems. Moore and Seiberg, in their work motivated by seeking a proof of the Verlinde conjecture [2], formulated an algebraic approach to RCFT's [3]. They refined the duality notations in the seminal work of BPZ [1] and obtained a whole set of polynomial equations for the duality data (see also [15] for a rigorous approach). Obviously, these data do not determine the conformal field theory completely: there are perhaps countably many models corresponding to a solution

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