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Quantization via Real Polarization of the Moduli Space of Flat Connections and Chern-Simons Gauge Theory in Genus One

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Abstract. We study the quantization of the moduli space of flat connections on a surface of genus one, using the real polarization of this space described in [10]. The quantum wave functions in this formalism are exponential functions supported along the integral fibres of the polarization. The space of wave functions obtained in this way is isomorphic to a space of theta functions. We use our construction to construct part of what may be a topological field theory in genus one, and to compute the associated invariants of some three manifolds. These computations agree with those of Witten [12], but the invariants are expressed as sums of quantities computed at a discrete set of connections with curvature concentrated on a link in the three manifold. A similar prescription is used to produce knot invariants.

I. Introduction

In [10] we showed that the moduli space $\bar{S}_g$ of flat $SU(2)$ connections on a two-manifold Σ^g of genus g possessed, in addition to the standard Kähler polarizations, a real polarization. One motivation for this study was that the quantization of this system in this real polarization may give some new insight into the structure of the conformal field theory related to this system, and may be a useful method to study Witten's quantum field theoretic interpretation of the Jones polynomial, with which it is intimately connected.

In this paper, we continue this program by studying the quantization of this system, where $SU(2)$ is replaced by any compact Lie group G, on a two-manifold of genus one. In this case the quantization may be carried out explicitly, and indeed is almost trivial. Nonetheless it exhibits many of the expected properties of topological quantum field theory, as axiomatized by Atiyah, and gives a number of other results. These are briefly as follows.

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