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Generalized Chiral Potts Models and Minimal Cyclic Representations of $U_q(\hat{\mathfrak{gl}}(n, \mathbf{C}))$

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Abstract. We present for odd N a construction of the N^{n-1}-state generalization of the chiral Potts model proposed recently by Bazhanov et al. The Yang–Baxter equation is proved.

1. Introduction

The discovery of the chiral Potts model [1–4] opened a new phase in the theory of Yang–Baxter equations (YBE). It gave the first example of an R matrix (=solution to YBE) whose spectral parameters live on an algebraic variety other than $\mathbf{P}^1$ or an elliptic curve. Through the latest developments [5–8] it has become apparent that quantum groups at roots of 1 should lead to this type of R matrices. Because of the technical complexity, this program has been worked out so far only in a few simple examples. Besides the chiral Potts model, which is related to $U_q(\hat{\mathfrak{sl}}(2, \mathbf{C}))$, these are the cases corresponding to $U_q(\hat{\mathfrak{sl}}(3, \mathbf{C}))$ ([7] for $q^3 = 1$, [9] for $q^4 = 1$) or $U_q(A_2^{(2)})$ [8]. In a recent paper [10] Bazhanov et al. proposed a generalization of the chiral Potts model related to N^{n-1} dimensional irreducible representations of $U_q(\hat{\mathfrak{sl}}(n, \mathbf{C}))$ at $q^N = 1$. The aim of this paper is to give a proof to their conjecture.

Let us formulate the problem more precisely. Throughout the paper we fix a primitive N^{th} root of unity q, with N an odd integer $\geqq 3$. We shall deal with a Hopf algebra $\tilde{U}_q$ (essentially the quantum double of a "Borel" subalgebra of $U_q(\hat{\mathfrak{gl}}(n, \mathbf{C}))$) [8]. As an algebra $\tilde{U}_q$ is a trivial extension of $U_q(\hat{\mathfrak{gl}}(n, \mathbf{C}))$ by central elements, with the comultiplication being twisted by them. In this paper

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