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Quantum Group Invariants and Link Polynomials

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Abstract. A general method is developed for constructing quantum group invariants and determining their eigenvalues. Applied to the universal R-matrix this method leads to the construction of a closed formula for link polynomials. To illustrate the application of this formula, the quantum groups $U_q(E_8)$, $U_q(so(2m+1))$ and $U_q(gl(m))$ are considered as examples, and corresponding link polynomials are obtained.

1. Introduction

Quantum groups [1, 2] play a fundamental role in the theory of integrable models [3], conformal field theory [4], and the classification of knots and links [5–10]. It appears possible that they may provide the key to understanding the intimate relationship between integrable systems and knot theory [8, 9], and even the more basic problem of why integrable models exist [8].

It is well known that to a large extent the study of a physical theory involves the exploration of its symmetries. In particular, it is desirable to determine the invariants of the symmetry algebra of the theory, which usually correspond to certain physical observables. Quantum groups arise as underlying symmetries of integrable lattice models and conformal field theory, and their invariants therefore are of crucial importance to the understanding of these problems. The quantum group invariants, being quantum analogs of the Gelfand invariants of ordinary Lie algebras, are also extremely useful for characterizing quantum group representations.

In this paper, we will develop a general method for constructing quantum group invariants and determining their eigenvalues. As a natural application we apply the method to Drinfeld's [2] universal R-matrix to obtain certain quantum group invariants, which in turn enable us to construct a closed formula for link polynomials. Our formula agrees with that of Reshetikhin [10] obtained through a completely different approach, but is more explicit, and our derivation is also more straightforward. To illustrate how the general formula works, we study as