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A Semi-Classical Trace Formula for Schrödinger Operators

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Abstract. Let $S_\hbar = -\hbar\Delta + V$ on $\mathbb{R}^n$, with V smooth. If $0 < E^2 < \liminf V(x)$, the spectrum of $S_\hbar$ near E^2 consists (for $\hbar$ small) of finitely-many eigenvalues, $\lambda_j(\hbar)$. We study the asymptotic distribution of these eigenvalues about E^2 as $\hbar \to 0$; we obtain semi-classical asymptotics for

$$\sum_j f\left(\frac{\sqrt{\lambda_j(\hbar)} - E}{\hbar}\right)$$

with $\hat{f} \in C_0^\infty$, in terms of the periodic classical trajectories on the energy surface $B_E = \{|\xi|^2 + V(x) = E^2\}$. This in turn gives Weyl-type estimates for the counting function $\#\{j;\ |\sqrt{\lambda_j(\hbar)} - E| \leqq c\hbar\}$. We make a detailed analysis of the case when the flow on B_E is periodic.

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1. Introduction

Consider the Schrödinger operator $S(\hbar) = -\hbar^2\Delta + V$ on $\mathbf{R}^n$, where $V \in C^\infty(\mathbf{R}^n)$, $V > 0$. If $\underline{V}_\infty = \liminf_{|x| \to \infty} V(x)$, the intersection

$$\mathrm{Spec}\, S(\hbar) \cap (-\infty, \underline{V}_\infty)$$

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