Commun. Math. Phys. 136, 501–517 (1991)

Communications in
Mathematical
Physics

A Kinetic Equation with Kinetic Entropy Functions for Scalar Conservation Laws*

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Received June 19, 1990; revised form August 7, 1990

Abstract. We construct a nonlinear kinetic equation and prove that it is well-adapted to describe general multidimensional scalar conservation laws. In particular we prove that it is well-posed uniformly in ε – the microscopic scale. We also show that the proposed kinetic equation is equipped with a family of kinetic entropy functions – analogous to Boltzmann's microscopic H-function, such that they recover Krushkov-type entropy inequality on the macroscopic scale. Finally, we prove by both – BV compactness arguments in the multidimensional case and by compensated compactness arguments in the one-dimensional case, that the local density of kinetic particles admits a "continuum" limit, as it converges strongly with $\varepsilon \downarrow 0$ to the unique entropy solution of the corresponding conservation law.

1. Introduction

Consider the scalar multi-dimensional conservation law

$$\frac{\partial}{\partial t}[u(x,t)] + \sum_{i=1}^{d} \frac{\partial}{\partial x_i}[A_i(u(x,t))] = 0, \quad (x,t) \in R_x^d \times R_t^+, \quad A_i(\cdot) \in C^1, \tag{1.1}$$

with given initial conditions $u(x,t=0) = u_0(x)$. We are concerned here with a Boltzmann-like kinetic equation which describes (1.1), as its microscopic scale, $\varepsilon > 0$, tends to zero. To this end we introduce a scalar function, $f_\varepsilon(x,v,t)$, which can be viewed as a microscopic description for the density of particles located at

* Research was supported in part by the National Aeronautics and Space Administration under NASA Contract No. NAS1-18605 while the authors were in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665. Additional support for the second author was provided by U.S.-Israel BSF Grant No. 85-00346. Part of this research was carried out while the first author was visiting Tel-Aviv University