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The Algebra of Weyl Symmetrised Polynomials and Its Quantum Extension

I. M. Gelfand* **and D. B. Fairlie**,*****

Harvard University, Cambridge, MA 02138, USA

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Abstract. The Algebra of Weyl symmetrised polynomials in powers of Hamiltonian operators P and Q which satisfy canonical commutation relations is constructed. This algebra is shown to encompass all recent infinite dimensional algebras acting on two-dimensional phase space. In particular the Moyal bracket algebra and the Poisson bracket algebra, of which the Moyal is the unique one parameter deformation are shown to be different aspects of this infinite algebra. We propose the introduction of a second deformation, by the replacement of the Heisenberg algebra for P, Q with a q-deformed commutator, and construct algebras of q-symmetrised Polynomials.

Introduction

In the first section of this article we present an account of some ideas, dating back to the pioneering work of Hermann Weyl which generalise the algebra of functions on a classical phase space to a space co-ordinatised by canonically quantised operators. These ideas have surfaced from time to time in the literature [1, 2, 3] and are now again very much alive in considerations of area preserving diffeomorphisms of two-dimensional manifolds, geometric quantisation and large N limits of $SU(N)$ [4, 5, 6]. The fundamental algebraic structure which we study is the algebra of symmetrised, averaged polynomials in P and Q, where P, Q satisfy the canonical commutation relations of the Heisenberg algebra;

$$PQ - QP = i\lambda. \tag{1}$$

Here we take λ as a numerical constant, but bear in mind that for possible novel

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** On Research Leave from the University of Durham, U.K. (Present Address)

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