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Lie Algebra Cohomology and $N=2$ SCFT Based on the GKO Construction

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Dedicated to Professor Noboru Tanaka on his sixtieth birthday

Abstract. We interpret $N=2$ superconformal field theories (SCFTs) formulated by Kazama and Suzuki via Goddard–Kent–Olive (GKO) construction from a viewpoint of the Lie algebra cohomology theory for the affine Lie algebra. We determine the cohomology group completely in terms of a certain subset of the affine Weyl group. We find that this subset describing the cohomology group can be obtained from its classical counterpart by the action of the Dynkin diagram automorphisms. Some algebra automorphisms of the $N=2$ superconformal algebra are also formulated. Utilizing the algebra automorphisms, we study the field identification problem for the branching coefficient modules in the GKO-construction. Also the structure of the Poincaré polynomial defined for each $N=2$ theory is revealed.

1. Introduction

Recent progress in two dimensional conformal field theories (CFTs) have revealed rich structures contained in the non-perturbative descriptions of the field theory, or presented us much variety allowed in the field theory. Among them, an important class of the CFTs are the so-called the rational CFTs [Ve, MS1]. The common property we note for the rational CFTs is that these theories can be constructed via Goddard–Kent–Olive (GKO) [GKO] construction using a suitable affine Lie algebra pair $(\hat{\mathfrak{g}}, \hat{\mathfrak{h}})$ with $\hat{\mathfrak{g}} \supseteq \hat{\mathfrak{h}}$. In this formulation of the CFTs, it is known that a suitable choice of $\hat{\mathfrak{g}}$ and $\hat{\mathfrak{h}}$ realizes the CFT with higher symmetry.

Recently Kazama and Suzuki [KS] showed that the $N=2$ superconformal field theory (SCFT) [BFK] [DPZ] [Na] can be constructed through the GKO

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